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Testimony on Montana's English Language Arts Standards:
Why Montana Should Replace its Common Core-Based Standards with Rigorous Standards

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Overview of Testimony: Thank you for the opportunity to testify to the deficiencies in Montana's current standards for English language arts (ELA). I first describe Common Core's Validation Committee, on which I served from 2009-2010. I then comment on Montana's current ELA standards, which are identical to Common Core's ELA standards. I offer recommendations that support implementation of the bill before the legislature.

My Credentials: I am professor *emerita* at the University of Arkansas, where I held the 21st Century Chair in Teacher Quality until my retirement in 2012. I served as Senior Associate Commissioner in the Massachusetts Department of Elementary and Secondary Education from 1999-2003, where I was in charge of developing or revising all the state's K-12 standards, teacher licensure tests, and teacher and administrator licensure regulations. I served on the Massachusetts Board of Elementary and Secondary Education from 2006-2010, on the National Mathematics Advisory Panel from 2006-2008, and on the Common Core Validation Committee from 2009-2010. I was one of the five members of the Validation Committee who did not sign off on the standards as being, rigorous, internationally competitive, or research-based.

I was also editor of the premier research journal, *Research in the Teaching of English*, published by the National Council of Teachers of English, from 1991 to 1997. I have published extensively in professional journals and written several books.

In recent years, I have testified before many legislative committees on the flaws in Common Core's standards and how a state can strengthen the education of all its students, as did the Bay State, by using academically rigorous standards for K-12 students and their teachers. Among the states in which I have testified or to which I have submitted invited testimony in writing are: Alabama, Alaska, Arkansas, California, Colorado, Connecticut, Georgia, Kansas, Kentucky, Michigan, Missouri, New Hampshire, North Carolina, North Dakota, Ohio, South Carolina, and West Virginia.

Development of Common Core's Standards
Common Core's K-12 standards were created by three private organizations in Washington, DC and did not emerge from a state-led process, as is often claimed. Nor were the people who wrote the standards qualified to write K-12 standards. The Validation Committee that was created to put the seal of approval on their work was unable to fulfill its charge. As a result, Common Core's standards and the tests based on them are not academically rigorous or valid.

Who were the standards writers and what were their qualifications? In the absence of official information from the three private organizations themselves, it seems likely that Achieve, Inc. and the Bill and Melinda Gates Foundation, which funded the project, selected most of the key personnel to write the college-readiness standards. Almost all the members of the Standards Development Work Groups that developed the high school-level standards were on the staff of Achieve, Inc. and three other test/curriculum development companies—American College Testing (ACT), America's Choice (a for-profit project of the National Center on Education and the Economy, also known as NCEE), and the College Board (CB). This crucial committee did not include any high school mathematics or English teachers.

The absence of relevant professional credentials in the two standards-writing teams helps to explain the flaws in these standards. The "lead" writers for the ELA standards, David Coleman and Susan Pimentel, had never taught reading or English in K-12 or at the college level. Neither has a doctorate in English. Neither has ever published serious work on K-12 curriculum and instruction. Neither has a reputation for literary scholarship or research in education. At the time they were appointed, they were virtually unknown to English educators.

The three lead standards writers in mathematics were as unknown to K-12 educators as were the ELA standards writers. None of the three standards writers in mathematics had ever developed K-12 mathematics standards that had been used—or used effectively. The only member of this three-person team with teaching experience, (consisting of 2 years teaching mathematics at the middle school level) Phil Daro, had majored in English as an undergraduate.

Who recommended these people as standards writers and why, we still do not know. No one in the media commented on their lack of credentials for the task they had been assigned. Indeed, no one in the media showed the slightest interest in the qualifications of the standards writers. Nor did the media comment on the low level of college readiness they worked out for high school.

Who were members of the Validation Committee? The federal government did not fund an independent group of experts to evaluate the rigor of the standards, even though it expected the states to adopt them (and still does). Instead, the private organizations in charge of the project created their own Validation Committee (VC) in 2009. The VC contained almost no academic experts in any area; most were education professors or associated with testing companies, from here and abroad. There was only one mathematician on the VC—R. James Milgram—although there were many people with graduate degrees in mathematics education, appointments in an education school, and/or who worked chiefly in teacher education. I was the only nationally recognized expert on English language arts standards by virtue of my work in Massachusetts and for Achieve, Inc.'s high school exit standards in its American Diploma Project.

Professor Milgram and I did not sign off on the standards because they were not internationally competitive (benchmarked), rigorous, or research-based. Despite our repeated requests, we did not get the names of high-achieving countries whose standards could be compared with Common Core's standards. Nor did the standards writers offer any research evidence to defend their omission of the mathematics standards needed for STEM careers, their de-emphasis on reading, their division of reading texts into "information" and "literature," their experimental approach to teaching Euclidean geometry, their deferral of the completion of Algebra I to grade 9 or 10, or their claim that informational reading instruction in the English class leads to college readiness. Nor did they offer evidence that Common Core's standards meet entrance requirements for *most colleges and universities* in this country or elsewhere.

Flaws in Montana's Current ELA Standards (see Appendix A)

1. Most of Montana's ELA standards are content-free skills. Most of the sentences that are presented as reading and literature standards are best described as skills or strategies. They point to no particular level of reading difficulty, little cultural knowledge, and few intellectual objectives. They point to no list of recommended authors or works. Nor do they require students to read high school-level texts in high school. They do not prepare students for college work, a career, or active citizenship in an English-speaking country.

2. Montana's ELA standards stress writing more than reading at every grade level—to the detriment of every subject in the curriculum. There are more writing than reading standards and objectives at almost every grade level, a serious imbalance. This is the opposite of what an academically sound reading/English curriculum should contain, as suggested by a large and consistent body of research on the development of reading and writing skills. The foundation for good writing is good reading. Students should spend far more time in and outside of school reading than writing in order to improve reading (and writing) in every subject of the curriculum.

3. Montana's writing standards are developmentally inappropriate at many grade levels. Adults have a much better idea of what "claims," "relevant evidence," and academic "arguments" are. Most elementary children have a limited understanding of these concepts and find it difficult to compose an argument with claims and evidence. It would not be easy for children to do so even if Montana's writing standards were linked to appropriate reading standards and prose models. But they are not. Nor does the document clarify the difference between an academic argument (explanatory writing) and opinion-based writing or persuasive writing, confusing teachers and students alike.

4. Montana expects English teachers to spend at least half of their reading instructional time at every grade level on informational texts. Montana lists 10 reading standards for informational texts and 9 standards for literary texts at every grade level, thus reducing literary study in the English class to less than 50%. There is no research that supports a decrease in literary study and an increase in informational reading in the English class as a way to improve college readiness.

5. Montana reduces opportunities for students to develop critical thinking. Critical, or analytical, thinking is developed in the English class when teachers teach students how to read between the lines of complex literary works. Analytical thinking is facilitated by the knowledge that students acquire in various ways because it cannot take place in an intellectual vacuum." As noted in a 2006 ACT report titled Reading Between the Lines: "complexity is laden with literary features." According to ACT, it involves "literary devices," "tone," "ambiguity," "elaborate" structure, "intricate language," and unclear intentions. Thus, reducing literary study in the English class in order to increase informational reading, in effect, retards college readiness.

6. Montana's standards are not "fewer, clearer, and deeper." They may appear to be few in number only because very different objectives or activities are often bundled incoherently into one "standard." As a result, they are not clearer or necessarily deeper. It is frequently the case that these bundled statements posing as standards are not easy to interpret and are poorly written.

Summary
(1) Montana's current ELA standards are NOT rigorous.
(2) Montana's standards are NOT internationally benchmarked and a curriculum based on them will not make its students competitive.
(3) There is NO research to support Montana's stress on writing instead of reading.
(4) There is NO research to support Montana's stress on informational reading instead of literary study in the English class.

Suggestions to Montana Legislators:
1. Develop rigorous, internationally benchmarked standards. For an interim period, adopt the highest-rated ELA standards in the country, such as California's, Indiana's 2006, Massachusetts 2001, or Texas 2008. These standards will be cheaper and easier for Montana teachers to use than Common Core's (what it now has). I have also provided free of charge a set of ELA standards, dated 2013, for any state to tailor as it wishes. It is the last reference on the list.

2. Ask your own engineering, science, and mathematics faculty and literary/humanities scholars to develop entrance exams (matriculation tests) for your own institutions of higher

education. Ask these faculty members to collaborate with mathematics and science teachers in Montana high schools in designing syllabi for the advanced mathematics and science courses in Montana high schools. Montana does not need bureaucrats in Washington DC to decide admission standards for Montana institutions. See Appendix B for issues in Montana's new mathematics standards.

3. Offer two different types of high school diplomas. Not all high school students want to go to college or can do the reading and writing required in authentic college coursework. Many have other talents and interests and should be provided with the opportunity to choose a meaningful four-year high school curriculum that is not college-oriented, as do students in most other countries.

4. Review and revise if needed all standards at least every five to seven years using identified Montana teachers, discipline-based experts in the arts and sciences, and parents. All assessments should also be reviewed by Montana teachers and discipline-based experts in the arts and sciences before the tests are given.

5. Restructure teacher and administrator training programs in Montana institutions of higher education to ensure that the teachers and administrators from these education schools have stronger academic credentials than they now have. Raising the floor for all children should be our primary educational goal, not closing demographic gaps among groups of children. The only finding from education research on teacher effectiveness is that effective teachers know the subject matter they teach. We need to raise the academic bar for every prospective teacher and administrator admitted to an educator training program in an education school. That is the first step in raising student achievement in this country.

References

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Sandra Stotsky. An English Language Arts Curriculum Framework for American Schools. http://alscw.org/news/?p=524

Appendix A. Flaws in Common Core's and Montana's ELA Standards

I. Missing Standards

1. No standard on the history of the English language.
2. No standard on British literature/authors aside from study of one Shakespeare play.

3. No standard on authors from the ancient world, especially classical Greece and Rome.

II. Overall Deficits

1. Standards are not real academic standards but processes or skills. See below.
2. Standards stress writing, not reading. Contradict 100 years of reading research and (more recent) prose model research in English. Good writers are first good readers.
3. Standards stress reading informational texts, not complex literary texts, for college readiness; no support from education research.
4. Standards foster little development of critical thinking; no research in cognitive psychology showing it is developed by reading informational texts in the English class.
5. Standards reduce literary study in the K-12 English class (only 9 of 19 reading standards address literary study at each grade level); also reduce vocabulary growth because older complex literary works feature larger non-technical vocabularies.
6. Document provides no selective lists of recommended authors, literary movements, or literary periods or traditions for classroom curriculum development or state assessment.
7. Standards document violates local control of curriculum by reducing literary study and requiring more "informational" texts at every grade level in the English/reading class.

III. Badly written, unclear "standards," not fewer, clearer, deeper true standards

For example, a literature "standard" for grades 9/10 asks students to: "determine a theme or central idea of a text and analyze in detail its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text."

This poorly constructed sentence jumbles at least three different activities: determining a theme, analyzing its development, and objectively summarizing a complete text. Moreover, it is not a true standard because it can be applied to *Moby-Dick* or to *The Three Little Pigs* It does not address literary knowledge, literary history, or a specific reading level.

Compare to an example of a true ELA standard, in California's pre-2010 standards for 11/12:

3.7 Analyze recognized works of world literature from a variety of authors:

a. Contrast the major literary forms, techniques, and characteristics of the major literary periods (e.g., Homeric Greece, medieval, romantic, neoclassic, modern).
b. Relate literary works and authors to the major themes and issues of their eras.

Or an example of a true ELA standard, in Massachusetts' pre-2010 standards for grades 9/10:

16.11: Analyze the characters, structure, and themes of classical Greek drama and epic poetry.

IV. Inappropriate literacy standards for study of history.

History study requires the use of such skills as contextualization, sourcing, and corroboration. These skills differ from those used in literary analysis and are not in Common Core.

Mark Bauerlein and Sandra Stotsky. How Common Core's ELA Standards Place College Readiness at Risk. Pioneer Institute White Paper # 89. http://pioneerinstitute.org/download/how-common-cores-ela-standards-place-college-readiness-at-risk/

Ralph Ketcham, Anders Lewis, and Sandra Stotsky. Imperiling the Republic: The Fate of U.S. History Instruction under Common Core. Pioneer Institute White Paper #121. http://pioneerinstitute.org/featured/study-common-core-ela-standards-will-further-harm-u-s-history-instruction/

Appendix B. Missing, Delayed, or Muddled Topics in Common Core's Math Standards

R. James Milgram and Ze'ev Wurman, Stanford University

Standard Algorithms in K to Grade 7: In mathematics standards, the words used to define the level to which students are to learn mathematical procedures or skills are *proficiency, mastery,* and *automaticity*, in that order However, *automaticity* and *mastery* never appear in Common

Core's mathematics standards (CCMS). In fact, *proficiency* appears with one exception only in the chapter *Standards for Mathematical Practice*, and only in the phrase "mathematical proficiency" or "mathematically proficient student."

The only word used in CCMS that could be interpreted as meaning one of those three words is "fluency." But Common Core embeds its meaning in the phrase "procedural fluency" (defined as "skill in carrying out procedures flexibly, accurately and appropriately") without explaining the kind of procedures students would carry out in this manner or the level of skill they should reach. Nor does it use the phrase more than once: it almost always uses "fluency with" or "fluently ... using" as a substitute.

Thus we find no requirement in CCMS that students reach the level of automaticity:

- for addition and subtraction with the standard algorithms or any other algorithms,
- for multiplication with the standard algorithm or any other algorithm, and
- for division with the standard long division algorithm or any other division algorithm.

Automaticity is the expectation for these algorithms in most if not all high-achieving countries. But not only is automaticity with the standard algorithms of arithmetic not required in CCMS, automaticity is also not required for any core skill or procedure mentioned in CCMS. Moreover, even mathematical errors occur in more advanced material, together with very careless writing.

On Ratios and Proportional Relationships in Grades 6 and 7

Grade 6: Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

   Comment: "nearly" does not correspond to ratio or rate in any way. At best it corresponds to a range of ratios, but the tools for handling such objects are not covered until college and require advanced calculus.

3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

   Comment: There is no indication of the size of the lawns or the amount of time it takes to mow each. Rather, the *assumption* is that they all take the same time to mow. Suppose some were 5000 square feet and some were 8000 square feet. We do not know the amount of time it takes to mow 8000 square feet compared to 5000 or if some lawns were steeply sloped and others level.

Grade 7: Analyze proportional relationships and use them to solve real-world and mathematical problems.

2. Recognize and represent proportional relationships between quantities.

   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

   Comment: In high-achieving countries, students are first given the definition: *two points in the coordinate plane, (a, b) and (c, d), are in a proportional relationship if and only if neither is (0, 0) and they both lie on a single straight line through the origin.* Presuming that a is non-zero, then writing $b = ra$ (so r = b/a), we see that d must equal rc for the two points to form a proportion. Without this starting point, the following sub-standards are completely confusing.

   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

   c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

   Comment: What is the graph of a proportional relationship? Any straight line through (0, 0)?

3. Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

   Comment: The given examples are all relatively trivial; the only non-trivial example (compound interest) is not included here or elsewhere in CCMS. Also, it is not clear why "percent error" occurs as an application example in grade 7, where a completely unmotivated formula for defining it (see below) is also given:

$$\frac{(\text{measured value}) - (\text{exact value})}{\text{exact value}} \times 100$$

   In addition, high-achieving countries introduce ratios and rates in grade 3 or 4 and students are expected to have mastered rate problems by grade 5 or 6.

We also find that:

- CC fails to teach decimals until grade 4, about two years behind high-achieving countries.
  CC fails to teach key geometrical concepts usually taught in K-7 (e.g., sum of angles in a triangle, isosceles and equilateral triangles). Sum of angles is taught in grade 8. Isosceles and equilateral triangles are taught in high school.
- CC excludes conversion between different forms of fractions: regular fractions, decimals, and percents. The word "conversion" does appear in five CCMS standards (noted below), but two deal with fractions and all are undemanding. Moreover, one of these

grade 7 standards has a minor mathematical error:

* The grades 4 and 5 standards (4.MD.1 and 5.MD.1) ask for conversion within a single system of units, e.g. feet to inches and centimeters to meters.

* Standard 6.RP.3d asks students to "Use ratio reasoning to convert measurements units."

* Standard 7.NS.2d asks students to "convert a rational number to a decimal using long division" but only as a method to "know that the decimal form of a rational number terminates in 0's or eventually repeats." Conversion is only an afterthought in this standard, which addresses irrational numbers.

> **Comment:** The phrase "the decimal form of a rational number terminates in 0's or eventually repeats" is redundant since *eventually repeats* includes terminating in 0. More important, this phrase should be a separate standard and not look like an afterthought. The key point of 7.NS.2d should have been that not every real number is the decimal expansion of a fraction. In other words, the real numbers properly contain the rational numbers.
>
> Also, Standard 7.NS.2d provides no FINITE algorithm for adding or multiplying real numbers, so it is technically meaningless. The process of "conversion" must be discussed more carefully.

* 7.EE.3, a 55-word long standard focused on solving multi-step problems with rational numbers, mentions "convert between forms as appropriate" among its multiple clauses almost as an afterthought.

- CC fails to teach prime factorization. Consequently, it does not teach least common denominators or greatest common factors, although "least common multiple" and "greatest common factor" are mentioned in Standard 6.NS.4 with a puzzling and completely unmotivated example:

> Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).

Since prime factorization is not discussed in CCMS, general methods for determining least common denominator or greatest common factor are not available to students. All they can reasonably be asked to do is to (laboriously) work out specific examples.

- CC omits teaching of compound interest and the formula for calculating it:

$$\frac{x^{n+1}-1}{x-1} = 1 + x + x^2 + \cdots + x^n.$$

This is a grade 7 or 8 topic in high-achieving countries, and was a grade 7 topic in previous California standards. CCMS provides a standard for this kind of problem in high school (Standard A-SSE.4), but too late and without sufficient background information and explanation:

> Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

Algebra 1: Missing components needed for Algebra 1 or beyond

1. Division of monomials and polynomials with remainder. Indeed, there is only one mention of polynomial remainders. It occurs on page 64 and refers to the simplest possible case, "the remainder theorem," which determines the remainder on dividing by x - a.
2. Derivation and understanding of the properties of slopes of parallel and perpendicular lines (that could
have been done easily using CC's formulation of geometry in terms of Euclidean transformations.

$$\frac{(ax+b)}{(cx+d)(ex+f)} = \frac{r}{(cx+d)} + \frac{s}{(ex+f)}$$

3. Manipulation and simplification of rational expressions. In particular, the basic property (partial fraction decomposition) for arbitrary (a, b), with its key applications to graphing and understanding rational functions, as well as basic preparation for pre-calculus, calculus, and more importantly for the solutions of differential equations in engineering and the sciences.[1]
4. Multi-step problems with linear equations and inequalities
5. Multi-step problems using all four operations with polynomials
6. Multi-step problems involving manipulation of rational expressions
7. Solving two (or more) linear inequalities in two variables and sketching the solution sets
8. Basic addition and half angle formulas for sin and cosine.
9. Any preparation for limits.
10. Almost no development of the standard properties of ellipses, hyperbolas, and parabolas, such as the existence and properties of the foci and directrix.

Geometry: Some missing key topics

Properties of triangles and circles: Students should know that:

- All three perpendicular bisectors of a triangle always intersect at a single point.
- Every triangle is circumscribed by a unique circle with a center at the intersection point of the three perpendicular bisectors of the edges.
- Every right triangle has the center of the circumscribing circle on its hypotenuse.
- The angle subtended by an arc on the circle (the angle obtained by drawing the two lines from the center to the ends of the arc) is twice the angle subtended by the ends of the arc and any point on the circle which is in the interior of the complement of the arc.

Issues with CCMS geometry. The geometry standards are very prescriptive, explaining exactly how they want the subject to be taught. The chosen method is non-standard and not validated by research. Indeed, some 35 years ago, the method was adopted in the former Soviet Union for the

[1]. William McCallum, a lead writer of the CCMS, co-authored a college textbook *Harvard Calculus* that also omitted partial fraction decomposition.

most advanced students but rapidly abandoned because it simply didn't work. CCMS requires geometry to be based on the properties of "the Euclidean Group" –the set of transformations of the coordinate plane consisting of reflections about any straight line, rotations through any angle with the center at any point in the plane, as well as these rotations and reflections followed by a translation.

For example, a key grade 8 geometry standard (Standard 8.G.2) is incomprehensible as written, for both students and teachers:

> Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Teachers have never seen anything like it, and students will wonder, "I've always heard that two figures are congruent if they have the same shape and size. How does 8.G.2 relate to this?" To add further confusion to the story, on page 64 of CCMS we find:

> "For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures."

Teachers now have to discuss "measures," that, somehow, congruence doesn't change, and that we have to somehow show that the equality of these measures ensures that two triangles with these measures are congruent. This is so advanced in reasoning and logic that only an unusual student in K-12 will have some idea of what this means. And in Standard 8.G.4, we find something worse:

> "Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them."

Up to this point both students and teachers have understood that two figures are similar "if they have the same shapes but not, necessarily, the same size." What do *dilations* have to do with this? We don't know and never find out. More disturbing, nowhere in CCMS are dilations ever defined. Thus it is not surprising that even advanced students in the countries of the USSR, among the highest-achieving countries in the world in mathematics, were unable to handle this approach in K-12.

Algebra II: Some key missing topics

1. Writing quadratic polynomials in two or three variables as sums or differences of perfect squares. (KEY for the study of conic sections, which, in turn, underlies a massive amount of the preliminary material in all STEM areas.)
2. Detailed study of surfaces of revolution coming from quadratic polynomials as described above. In particular, the focus here should be on parabolic mirrors and their applications.
3. Introduction of the foci and the directrix for conics and their applications to parabolas and parabolic mirrors, as well as for ellipses and elliptic surfaces with applications to things

like whispering galleries and Kepler's laws.
4. Definition and implications of the eccentricity for conic sections.
5. Structure of logarithms to base 10, e, or general base, $b > 0$. Conversion between bases, calculation of explicit values in simple cases.

Algebra II: Missing components needed for Calculus

- Composite functions (for example functions of the form $f(g(x))$ if the domain of f contains the range of g). There is one and only one mention of composite functions (F-BF.4b) and then only in the context of one of the most special cases possible.
- Combinations and permutations. (There is only one mention of them, and only in a (+) standard on page 82 of CCMS, S.CP.9. But they and the associated binomial coefficients form the basis for virtually all combinatorial results that are used in many, if not most, real-world applications of mathematics.)
- Finite and infinite arithmetic and geometric sequences
- Mathematical induction

All four topics above are quite ``formal'' in line with the overly formal treatment of algebra in Common Core's Standards. But they are much more ``realistic'' in terms of the actual needs of students wishing to major in any technical area in college.

Pre-calculus and/or Algebra II, Trigonometry: Some key missing topics

1. Partial fraction decomposition of relatively simple rational functions and their graphs. The partial fraction decomposition obtained in (Algebra 1, Missing components, item 3) has r and s determined as the solutions of the two linear equations in two unknowns, $er + cs = a$, and $fr + ds = b$. (One can always find r and s as long as $cx + d$ is not a constant multiple of $ex + f$, since this implies that the determinant $cf - ed \neq 0$. So this system of linear equations has one and only one solution.)

   This is one of the key applications of the systems of linear equations that are supposed to be studied in Algebra I or earlier, to say nothing of the addition formula for fractions.) The other key application, *linear regression* (determining the regression line of a set of data points in the plane) is far too advanced for high school mathematics since it requires multi-variable calculus.

2. Graph functions in polar coordinates. Key examples (all standard topics in high-achieving countries):
   Circles written in the form $r = 2\cos(t)$,
   Cardioids $(2 + 2\cos(t) = r)$,
   Rose petal curves $(r = \sin(5t))$, and
   Lemniscates ($r^2 = 4\sin(2t)$).

Definition of vectors: All standards relating to vectors in CCMS are (+) topics, but they are confusing to the point that they contain mathematical errors. For example, consider Standard N-VM.1:

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).

   Comment: This standard confuses vectors (points in the coordinate plane that one adds coordinate-wise [(A, B) + (a,b) = (A+a, B+b)] and multiplies by a number (scales) via the rule c(A, B) = (cA, cB) -) with the field of (tangent) vectors on the plane, and this confusion continues in the remaining standards in this section, as the next example illustrates.

2. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

    Comment: Velocity means "velocity AT A POINT," which associates a tangent vector at the point in question to the point. In other words, velocity involves the field of tangent vectors, not the elements in a single vector space. What could it possibly mean to add velocities at *two different points?*

The meaning of "rigorous"

The academic level of the CCMS is dramatically lower than the level in high-achieving countries even though school administrators, self-described policy makers, as well as education school faculty generally describe CCMS as "rigorous." So, how did this word get to be used to describe CCMS?

In some cases, people using this term have had as their subjective reference the pre-CC standards in the weakest states, e.g., the 2003 Missouri or Wyoming mathematics standards. Many have said that CCMS are more rigorous than the state standards they replaced. But there may be a better explanation for the use of this term to describe CCMS.

The *Dictionary of Education Reform* (edglossary.org/rigor) offers the following definition of "rigor":

> "While dictionaries define the term as rigid, inflexible, or unyielding, educators frequently apply (the terms) rigor or rigorous to assignments that encourage students to think critically, creatively, and more flexibly. Likewise, they may use the term rigorous to describe learning environments that are not intended to be harsh, rigid, or overly prescriptive, but that are stimulating, engaging, and supportive."

Consequently, when ordinary parents hear the word "rigorous" used to describe CCMS, they are likely to misunderstand what the speaker actually means. It is not possible to describe CCMS as extremely thorough, careful, or even accurate, since many of the standards are undemanding, omit many key topics, are unclearly written, and even have mathematical errors. At best, education professionals who use "rigorous" to describe CCMS may well be saying that its standards promote "creativity and flexible thinking" (although they do not indicate how) and they may also be implying absolutely nothing about accuracy or intellectual demand.

On the other hand, for foundational mathematics, neither creativity nor flexibility is desired. We do not want students to decide what 37/7 *should* or *could* be. Rather, we want them to be able to say that 37/7 is 5 + 2/7, or 5(2\over 7), or perhaps 5.\overline (285714).